LOWELL HIGH SCHOOL
SUMMER REVIEW PACKET

For students entering Calculus

Name__________________

Period_________________

PLEASE DO ALL WORK ON THIS PACKET!!
DO NOT ATTACH EXTRA PAPERS

1. This packet is due on <u>the first day</u> of the school year.
2. All work must be shown on the packet.
3. Completion of this packet is worth one-half of a test grade. The test on the summer review material will be administered within the first week of school.
4. This work should take you approximately 8 hours, so you should plan accordingly.
5. If you are unable to do any of these problems, you may use the site of KHAN ACADEMY.

Complex Fractions

When simplifying complex fractions, there are different ways to simplify, two of which are shown below:

1. **work separately with the numerator and denominator, rewriting each with a common denominator, and then multiplying the numerator by the reciprocal of the denominator; or**
2. **multiply the entire complex fraction by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.**

Example:

1) $$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7(x+1)-6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-13}{x+1}}{\frac{5}{x+1}} = \frac{-7x-13}{x+1}\cdot\frac{x+1}{5} = \frac{-7x-13}{5}$$

OR: $$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\cdot\frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

2) $$\frac{\frac{-2}{x}+\frac{3x}{(x-4)}}{5-\frac{1}{x-4}} = \frac{\frac{-2(x-4)+3x^2}{x(x-4)}}{\frac{5(x-4)-1}{x-4}} = \frac{3x^2-2x+8}{x(x-4)}\cdot\frac{x-4}{5x-21} = \frac{3x^2-2x+8}{(x)(5x-21)} = \frac{3x^2-2x+8}{5x^2-21x}$$

OR:

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}}\cdot\frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

Composite Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

4. $f\left(g(-2)\right) =$ __________

5. $g\left[f(m+2)\right] =$ __________

6. $\dfrac{g(x+h) - g(x)}{h} =$ ______

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$**. Find each.**

7. $f\left[g(x-1)\right] =$ _______

8. $g\left[h(x^3)\right] =$ _______

Find $\dfrac{f(x+h) - f(x)}{h}$ **for the given function *f*.**

9. $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, also referred to as the zeros of the function, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$\underline{x - \text{int. } (Let\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1 \text{ or } x = 3$

$x - \text{intercepts } (-1, 0) \text{ and } (3, 0)$

$\underline{y - \text{int. } (Let\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept } (0, -3)$

Find the x and y intercepts for each.

10. $y = 2x - 5$ 11. $y = x^2 + x - 2$ 12. $y = x\sqrt{16 - x^2}$ 13. $y^2 = x^3 - 4x$

Interval Notation

14. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		 8

Solve each equation. State your answer in BOTH interval notation and graphically.

15. $2x - 1 \ge 0$ 16. $-4 \le 2x - 3 < 4$ 17. $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

18. $f(x) = x^2 - 5$ 19. $f(x) = -\sqrt{x+3}$ 20. $f(x) = 3\sin x$ 21. $f(x) = \dfrac{2}{x-1}$ 22. $f(x) = \dfrac{4}{\sqrt{2x-5}}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
y = $\sqrt[3]{x+1}$	Switch x and y
x = $\sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

23. $f(x) = 2x + 1$

24. $f(x) = \dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right) + 9 \qquad g(f(x)) = \frac{(4x+9)-9}{4}$$

$$= x - 9 + 9 \qquad = \frac{4x+9-9}{4}$$

$$= x \qquad = \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

25. $f(x) = \dfrac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

26. $f(x) = 9 - x^2,\ x \geq 0$ $\quad g(x) = \sqrt{9-x}$

<u>**Equation of a line**</u>

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

27. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

28. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

29. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

30. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \dfrac{5}{6}x - 1$.

31. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

<u>Radian and Degree Measure</u>

Note: In calculus, we always use radians, unless noted otherwise!

Use $\frac{180^{\circ}}{\pi\, radians}$ to convert from radians to degrees.

Use $\frac{\pi\, radians}{180^{\circ}}$ to convert from degrees to radians.

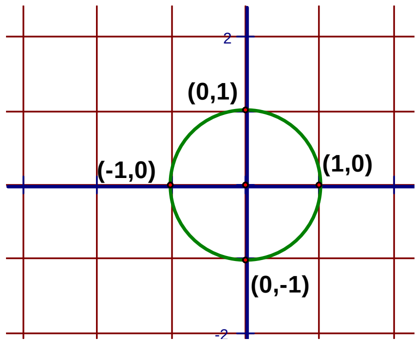
32. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

33. Convert to radians: a. 45° b. -17° c. 237°

<u>Unit Circle</u>

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90^{\circ} = 1$ $\cos\frac{\pi}{2} = 0$



34. $a.)\ \sin 180^{\circ}$ $b.)\ \cos 270^{\circ}$ $c.)\ \sin(-90^{\circ})$ $d.)\ \sin\pi$ $e.)\ \cos 360^{\circ}$ $f.)\ \cos(-\pi)$

35. **Without a calculator**, determine the <u>exact value</u> of each expression.

a) $\sin 0$ b) $\sin\frac{\pi}{2}$ c) $\sin\frac{3\pi}{4}$ d) $\cos\pi$ e) $\cos\frac{\pi}{3}$ f) $\cos\frac{3\pi}{4}$

g) $\tan\frac{7\pi}{4}$ h) $\tan\frac{\pi}{6}$ i) $\tan\frac{2\pi}{3}$ j) $\sec\frac{\pi}{3}$ k) $\csc\frac{5\pi}{4}$ l) $\cot\frac{\pi}{2}$

<u>Trigonometric Equations:</u>

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$.

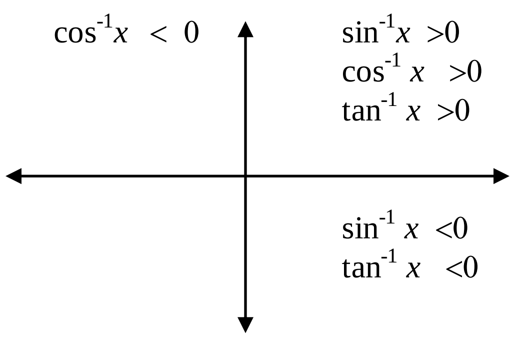
36. $2\cos x = \sqrt{3}$ 37. $\sin^2 x = \frac{1}{2}$ 38. $4\cos^2 x - 3 = 0$

<u>Inverse Trigonometric Functions:</u>

Recall: Inverse Trig Functions can be written in one of ways:

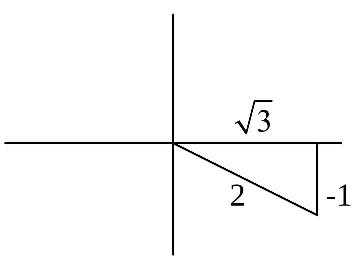
$\arcsin(x)$ $\sin^{-1}(x)$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = – $\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: y = – $\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

39. $y = \arcsin\frac{1}{2}$ (or $\sin^{-1}\frac{1}{2}$)

40. $y = \arctan 1$

41. $y = \arcsin\frac{-\sqrt{3}}{2}$

42. $y = \arccos(-1)$

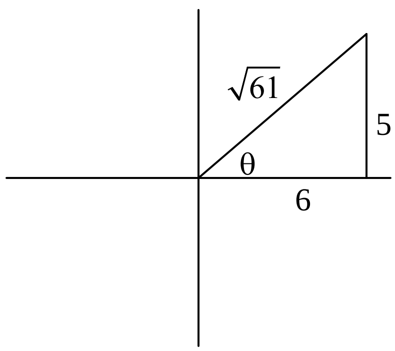
Example: Find the value without a calculator.

$\cos\left(\arctan\frac{5}{6}\right)$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Theorem.

Find the ratio of the cosine of the reference triangle. $\cos\theta = \frac{6}{\sqrt{61}}$



For each of the following find the value without a calculator.

43. $\tan\left(\arccos\frac{2}{3}\right)$

44. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

45. $\sin\left(\arctan\frac{12}{5}\right)$

46. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

47. Take a break – you deserve it!

48. Tell someone in your family that you love them and note their reaction.

Logarithms and Exponentials

$$y = \log_b x \text{ is equivalent to } x = b^y$$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

$\log_b 1 = 0$, ln1=0, $\log_b a = 1$, $\ln e = 1$

Because logarithms and exponentials are inverse functions of each other:

$\log_b(b^x) = x$, $\ln(e^x) = x$, $\mathrm{b}^{\log_b x} = x$, $e^{\ln x} = x$

49. Solve each exponential or logarithmic equation.

$a)$ $5^x = 125$ $b)$ $8^{x+1} = 16^x$ $c)$ $81^{\frac{3}{4}} = x$

$d)$ $8^{\frac{-2}{3}} = x$ $e)$ $\log_2 32 = x$ $f)$ $\log_x \frac{1}{9} = -2$

$g)$ $\log_4 x = 3$ $h)$ $\log_3(x+7) = \log_3(2x-1)$ i) $\log x + \log(x-3) = 1$

50. Expand each of the following using the properties of logs.

$a)\log_3 5x^2$ $\qquad b)\quad \ln\dfrac{5x}{y^2}$

51. Evaluate the following expressions.

a) $e^{\ln 3}$ b) $e^{(1+\ln x)}$ c) $\ln 1$ d) $\ln e^7$

b) e) $\log_3(1/3)$ f) $\log_{1/2} 8$ g) $e^{3\ln x}$

52. Expand: $(x+y)^3$

53. Solve for x. Show the work that leads to your solution.

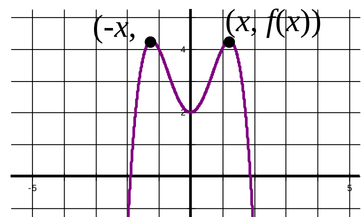
a) $\dfrac{x^4-1}{x^3}=0$ b) $(x-5)^2=9$ c) $2x+1=\dfrac{5}{x+2}$

d) $x^2-2x-15\le 0$ e) $|x-3|<7$ f) $27^{2x}=9^{x-3}$

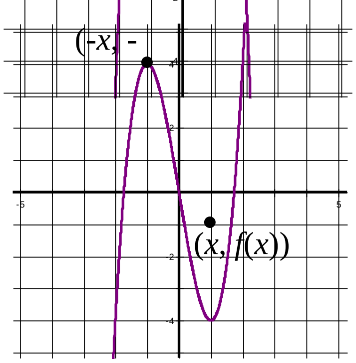
54. Rationalize the denominator.

(a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ (b) $\dfrac{1-x}{\sqrt{x}-1}$

SYMMETRY



If a function *f* satisfies *f*(-x) = *f*(x) for every number *x* in its domain, the *f* is called an even function. For example *f*(x) = $x^4 + 2x^2 + 7$ is an even function because *f*(-x) = $(-x)^4 + 2(-x)^2 + 7 = x^4 + 2x^2 + 7 = f(x)$
The geometric significance of an even function is that its graph is symmetric with respect to the y – axis.



If *f* satisfies *f* (-x) = -*f*(x) or every number in its domain, then *f* is called an odd function. For example, the function *f*(x) = $2x^3 + 7x$ is odd because
f (-x) = $2(-x)^3 + 7(-x) = -2x^3 - 7x = -(2x^3 + 7x) = -f(x)$
The graphs of an odd function is symmetric about the origin.

Determine algebraically whether each of the following functions is even, odd, or neither. Show all your work.

56.f(x) = $x^5 + x$

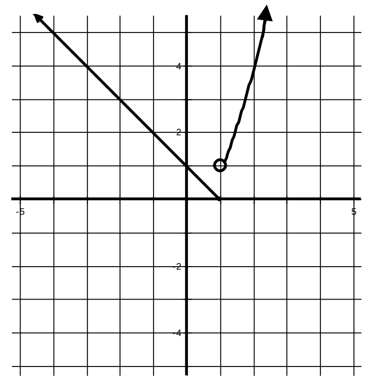
57. f(x) = $\frac{x}{x+1}$

58. f(x) = $\frac{x^2}{1+x^4}$

PIECE-WISE FUNCTIONS

A piecewise function is a function that is defined by different formulas in different part of their domains.

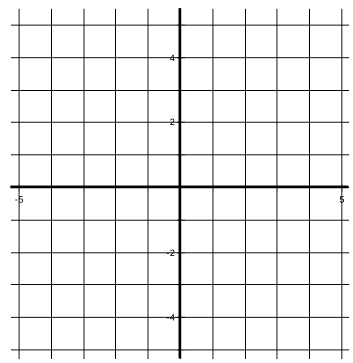
Example: $f(x) = \begin{cases} 1 - x & if\ x \le 1 \\ x^2 & if\ x > 1 \end{cases}$



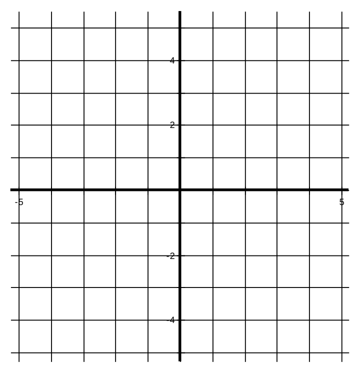
To sketch the graph of f(x), sketch in two parts $y_1 = 1 - x$ for all $x \le 1$
And $y_2 = x^2$ for all $x > 1$

You try.

59. $f(x) = \begin{cases} x + 2 & x < 0 \\ 1 - x & x \ge 0 \end{cases}$



60. $f(x) = \begin{cases} x + 2 & x \le 0 \\ x^2 & x > 0 \end{cases}$



SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

Show all work.

61. $3^{2x-3} = 81$

62. $2^{5-2x} = \frac{1}{2}$

63. $2^{4x+1} = \sqrt{2}$

64. $9^{2x-4} = \left(\frac{1}{27}\right)^{x-3}$

65. $\left(\frac{1}{32}\right)^{X-7} = \left(\frac{1}{8}\right)^{X-11}$

66. $\left(\frac{1}{4}\right)^{2-2X} = \left(\sqrt[3]{2}\right)^{3X+6}$

67. $\log_4 256 = x$

68. $\log_{\sqrt{3}} 27 = x$

69. $\ln e^7 = x$

70. $e^{\ln \sqrt{e}} = \text{x}$

71. $\frac{3\log 10}{2\ln e} = x$

72. $10^{\log 29} = 0$

73. $\log_3(2x-2) = 2$

74. $\ln 3x + \ln 3 = 3$

75. $\log_2 (x-1) + \log_2 (x+3) = 5$

76. log (4x + 22) – log (2x + 1) = 1

77. $\log_5 (x+3) - \log_5 x = 2$

78. $\log_6 50 = x$

79. $\log_2 25 = x$

80. $\log_{12} 24 = \text{x}$